

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

NAME \_\_\_\_\_

Math Analysis: Trig Identities-Ten Proofs

Verify the following identities:

1.  $\frac{\cos^2 \theta}{\sin \theta} + \sin \theta = \csc \theta$

$$\frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta} = \csc \theta$$

$$\frac{1}{\sin \theta} = \csc \theta$$

$$\csc \theta = \csc \theta \checkmark$$

2.  $\csc^4 \alpha - \cot^4 \alpha = \csc^2 \alpha + \cot^2 \alpha$

$$(\csc^2 \alpha + \cot^2 \alpha)(\csc^2 \alpha - \cot^2 \alpha) = \csc^2 \alpha + \cot^2 \alpha$$

$$(\csc^2 \alpha + \cot^2 \alpha)(1) = \csc^2 \alpha + \cot^2 \alpha$$

$$\csc^2 \alpha + \cot^2 \alpha = \csc^2 \alpha + \cot^2 \alpha \checkmark$$

3.  $(1 - \tan \beta)^2 = \sec^2 \beta - 2 \tan \beta$

$$1 - 2 \tan \beta + \tan^2 \beta = \sec^2 \beta - 2 \tan \beta$$

$$1 + \tan^2 \beta - 2 \tan \beta = \sec^2 \beta - 2 \tan \beta$$

$$\sec^2 \beta - 2 \tan \beta = \sec^2 \beta - 2 \tan \beta \checkmark$$

4.  $2 \sin^2 \theta - 1 = 1 - 2 \cos^2 \theta$

$$2(1 - \cos^2 \theta) - 1 = 1 - 2 \cos^2 \theta$$

$$2 - 2 \cos^2 \theta - 1 = 1 - 2 \cos^2 \theta$$

$$1 - 2 \cos^2 \theta = 1 - 2 \cos^2 \theta \checkmark$$

5.  $\frac{1 - \sin^2 \eta}{1 + \tan^2 \eta} = \cos^4 \eta$

$$\frac{\cos^2 \eta}{\sec^2 \eta} = \cos^4 \eta$$

$$\frac{\cos^2 \eta}{\frac{1}{\cos^2 \eta}} = \cos^4 \eta$$

$$\cos^2 \eta : \cos^2 \eta = \cos^4 \eta \checkmark$$

7.  $\frac{\sin \alpha + \tan \alpha}{1 + \sec \alpha} = \sin \alpha$

$$\frac{\sin \alpha + \frac{\sin \alpha}{\cos \alpha}}{1 + \frac{1}{\cos \alpha}} = \sin \alpha$$

$$\frac{\sin \alpha \left(1 + \frac{1}{\cos \alpha}\right)}{1 + \frac{1}{\cos \alpha}} = \sin \alpha$$

$$\sin \alpha = \sin \alpha \checkmark$$

9.  $\frac{1 + \cos \delta}{\sin \delta} = \csc \delta + \cot \delta$

$$\frac{1}{\sin \delta} + \frac{\cos \delta}{\sin \delta} = \csc \delta + \cot \delta$$

$$\csc \delta + \cot \delta = \csc \delta + \cot \delta \checkmark$$

6.  $\frac{\sin \theta}{\csc \theta} + \frac{\cos \theta}{\sec \theta} = 1$

$$\frac{\sin \theta}{\frac{1}{\sin \theta}} + \frac{\cos \theta}{\frac{1}{\cos \theta}} = 1$$

$$\sin \theta \cdot \sin \theta + \cos \theta \cdot \cos \theta = 1$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 = 1 \checkmark$$

8.  $\frac{1 - \tan^2 \lambda}{1 + \tan^2 \lambda} = 2 \cos^2 \lambda - 1$

$$\frac{1 - \tan^2 \lambda}{\sec^2 \lambda} = 2 \cos^2 \lambda - 1$$

$$\frac{1 - (\sec^2 \lambda - 1)}{\sec^2 \lambda} = 2 \cos^2 \lambda - 1$$

$$\frac{1 - \sec^2 \lambda + 1}{\sec^2 \lambda} = 2 \cos^2 \lambda - 1$$

$$\frac{2 - \sec^2 \lambda}{\sec^2 \lambda} = 2 \cos^2 \lambda - 1$$

10.  $\frac{\sec \phi \sin \phi}{\tan \phi + \cot \phi} = \sin^2 \phi$

$$\frac{\frac{1}{\cos \phi} \cdot \frac{\sin \phi}{1}}{\frac{\sin \phi}{\cos \phi} + \frac{\cos \phi}{\sin \phi}} = \sin^2 \phi$$

$$\frac{\frac{\sin \phi}{\cos \phi}}{\frac{\sin^2 \phi + \cos^2 \phi}{\sin \phi \cos \phi}} = \sin^2 \phi$$

$$\frac{\frac{\sin \phi}{\cos \phi}}{\frac{1}{\sin \phi \cos \phi}} = \sin^2 \phi$$

$$\frac{\sin \phi}{\cos \phi} \cdot \frac{\sin \phi \cos \phi}{1} = \sin^2 \phi$$

$$\sin^2 \phi = \sin^2 \phi \checkmark$$